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Resistive MHD Stability Theory

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1 Overview

2 The pressure flattening inside magnetic islands and the degradation of the confinement





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1 Overview

- 2 The pressure flattening inside magnetic islands and the degradation of the confinement
- 3 Tearing instability
- 4 Conclusion

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A biased picture of a Tokamak



A biased picture of a Tokamak



Magnetic islands (MHD perturbation) and resonance



A closed equilibrium magnetic field line $d\textbf{M} \times \textbf{B}_{_{eq}} = 0$ satisfies

$$q(r) = d\phi/d\theta = rB_{\phi}(RB_{\theta})^{-1} = m_{res. surf.}/n_{res. surf.} + q(r_s) = Cte$$

Thus, a closed line winds onto a rationnal magnetic surface.

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- Equilibrium: $\mathbf{B}_{\scriptscriptstyle{eq}} = B_{\phi} \hat{\phi} + B_{\theta} \hat{\theta}$ (Nested magnetic surfaces)
- A perturbation $\mathbf{B} = \widetilde{\mathbf{B}}(r, t) \exp i(m_{\text{pert}}\theta n_{\text{pert}}\phi)$ is resonant if $m_{\text{pert}}/n_{\text{pert}} = m_{\text{res. surf.}}/n_{\text{res. surf.}}$
- Any resonant pertubation $h = \tilde{\mathbf{h}}(r, t) \exp i(m_{\text{pert}}\theta n_{\text{pert}}\phi)$ satisfies $\nabla_{\parallel} h = B_{\text{eq}}^{-1} \mathbf{B}_{\text{eq}} \cdot \nabla h = 0$ $(\nabla_{\parallel} h = im_{\text{pert}} B_{\text{eq}}^{-1} B_{\theta,\text{eq}} / r(1 - n_{\text{pert}} m / m_{\text{pert}} n) h = 0.)$

Magnetic islands are resonant perturbations growing from low order rational equilibrium magnetic surfaces

[http://www.vacet.org/,NIMROD]





- Magnetic islands are <u>non local perturbations</u> of the equilibrium. Thus, low order rationnal surface are required (see drawing...)
- Large islands are seeded on low order rationnal surfaces $m_{\rm res.~surf.}/n_{\rm res.~surf.}=2/1$ or 3/2.
- Islands are helical structures => Symmetry $y \propto \theta m/n\phi$, $x \propto r r_s$

Growth of magnetic islands by current driven tearing instability





• Left: Mono-helicity simulation (only modes with m/n = 2 evolve)

Right: Full 3D simulation (global simulation: all the modes evolve)

No difference...

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Stability Diagrams

[from Biskamp, Nonlinear MHD, Cambridge (1993)]



Fig. 4.2. Stability diagram for kink modes in a straight tokamak for current profiles $j(r) = j_0(1 - r^2/a^2)^v$. The stable region is unshaded (from Wesson, 1978).



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- Left: The kink stability domain (ideal) is strongly constrained in tokamaks
- Right: Full 5D simulations using GKW

=> Islands generically grow in the stability domain (to avoid them further stability and discharge path analysis are required.)



[T.C. Hender et al, Nuc. Fusion 47 (2007)]



Sketch of the time evolution of the island size *w* of an island (power-ramp down experiment).

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- Tearing instability, MHD event,... can generate a magnetic island. <u>Neoclassical physics can amplify</u> it.
- When $w > w_{cri}$, Seed islands \Rightarrow Radially extended islands or NTMs
- Neoclassical amplification is linked to a reduction of the bootsrap current (not only: polarization effects also,...)

Impact of INTIVIS: path to disruptions

[From PhD thesis Alexandre Fil, AMU (2015)]





Figure 2.10: A schematic overview, showing the statistics of the sequence of events for 163(d) unintentional disruptions at JET during the period 2000 to 2010. The width of the connecting arrows indicates the frequency of occurrence with which each sequence took place (only those paths with an occurrence of >0.25% are shown). Note that the disruption process could start at any node (event) in the overview, which generally, but necessarily, flows from left to right. The labels correspond to those listed in tables 2.1 and 2.2.

Main types of physics problem	Label
General (rotating) n = 1 or 2 MHD	MHD
Mode lock	ML
Low q or $q_{95} \simeq 2$	LOQ
Edge q close to rational (> 2)	QED
Radiative collapse (Prad > Pin)	RC
Greenwald limit (nGW)	GWL
Strong pressure profile peaking	PRP
Large edge localized mode (ELM)	ELM
Vertical displacement event	VDE

Table 2.1: Examples of physics problems

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Impact of NTMs



[S. Gunter et al, Phys. Rev. Lett. 87 (2001)]



Reduction in energy confinement $\Delta W/W$ due to (3,2) NTMs on ASDEX Upgrade (same results in JET)

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- Path to disruptions, but not only...
- Degradation of the energy confinement $\propto w$ and/or $\propto \beta = \frac{\text{pressure}}{\text{magnetic energy}}$
- Existence of unexpected high confinement regimes at high $\beta_N\gtrsim$ 2.3, call FIR-NTM regimes

Metastability of NTMs



[R.J. Buttery et al, IAEA Conference, (2004 and 2008)]

• NTMs should be metastable in ITER: $eta \gg eta_{ ext{marg}}$

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Origin of seed islands ($w \lesssim w_{rri}$)

[S. Fietz et al,41st EPS Conf. on Plasma Physics (2014)]



Trigger Mechanisms of (2,1) NTMs in normalised (β , $v_{\rm toroidal}$) space in ASDEX Upgrade Tokamak

[A. Isayama et al, Plasma and Fusion Research 8 (2013)]



Onset of (2,1) NTMs in high β_p discharges in JT60U Tokamak.

In about 80% of the discharges, (2,1) NTM appear from a small amplitude without any noticeable triggering event. Turbulence might be a trigger of such NTMs [Muraglia et al, NF 2017]

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[PhD Thesis A. Poyé, AMU (2012)]



• Equilibrium => Island : Impossible in Ideal MHD because a change of field line topology requires the violation of local magnetic flux conservation (Frozen in Theorem)

•
$$\psi(x,y) = \psi_0(x) + \delta^2(t) \cos(k_1 y)$$
 with $k_1 = \frac{2\pi}{L_y}$ and
 $\mathbf{B}_{eq}(x) = \psi'_0(x) \mathbf{\hat{y}}, \ \mathbf{j}_{eq}(x) = \psi''_0(x) \mathbf{\hat{z}},$
 $\mathbf{B}(x,y,t) = \mathbf{\hat{z}} \times \nabla \psi(x)$

• Structure of a magnetic island: Separatrices, X-point, O-point. $w = 4\sqrt{\frac{\delta^2}{j_{eq}(0)}}$ is the width of the island



[L. Vermare et al, PPCF, 47, 1895 (2005)]

J. A. Snape et al, PPCF 54 085001 (2012]

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• Both, density n and electronic temperature T_e profile can be flattenned by the growth of a magnetic island if it is large enough...

Pressure flattening mechanism

 Hyp 1: no source in the vicinity of an island, thus $\nabla \cdot \mathbf{q}_e = 0$ $\nabla \cdot \mathbf{q}_e = -\nabla_{\perp} \cdot (n\chi_{\perp}\nabla_{\perp}T_e) - \nabla_{\parallel} \cdot (n\chi_{\parallel}\nabla_{\parallel}T_e)$ with $\chi_{\parallel} = \chi_{\parallel}(T_e) \propto T_e^{1/2} L_{\parallel}$ and $\chi_{\perp} = \chi_{\perp}^{\text{turbulence}} = \chi_{\perp} (T_e, \nabla T_e / T_e)$ • Hyp 2: The island size w satisfies $\chi_{\perp}/w^2 \ll \chi_{\parallel}L_{\parallel}^2$, $=> \nabla_{\parallel} T_e = \Delta_{\parallel} T_e = 0.$ In other words, if $w \gg w_c = \sqrt{\chi_\perp/\chi_\parallel L_\parallel}$ then $T_e = T_e(\psi)$ In fact, $abla_{\parallel} \sim L_{\parallel}^{-1} = k_1 w/L_s$, $w_c = (\chi_{\perp}/\chi_{\parallel})^{1/4} \sqrt{L_s/k_1}$ The flux accross the island is $0 = <\nabla \cdot \mathbf{q}_{\mathbf{e}} >_{\text{island}} = <\mathbf{q}_{\mathbf{e}} \cdot \mathbf{n} >_{\text{SED}} = T'_{\mathbf{e}}(\psi) < n\chi_{\perp}\nabla_{\perp}\psi >$ $= T_e(\psi) = Cte$ in a magnetic island if $w \gg w_c$ • For the density, we need $D_{\perp}/w^2 \ll D_{\parallel}L_{\parallel}^2$ but $D_{\perp}/D_{\parallel} \sim \chi_{\perp}/\chi_{\parallel} (m_i/m_e)^{lpha}$ => Temperature flattening occurs before density flattening

Degradation of the confinement



- Degradation of the confinement: Topology + parallel diffusion
 => The heat follows the separatrices (resistive layer) => Radial short circuit for the heat from the core to the edge
- The cylindrical Belt model: $\chi(r_{-} \le r \le r_{+}) = +\infty$ (Infinite conductivity in the vicinity of the island) ... see blackboard, [Chang et al, Nuc; Fus. 30 211 (1990)]
- $\Delta E_{\rm th}/E_{\rm th} = f(r_s)\frac{w}{a}$ with $f(y) = 4y^3$ for the belt model where n_0, χ_0 and Q_0 are supposed constants (f depends on island geometry and equilibrium density and temperatures)
- NTMs can have easily 10% of the radius. So

[Sauter et al, PPCF 52 025002 (2010)]

- Belt model at mid-radius, gives a degradation of 5%
- Belt model at 0.8a, gives a degradation of 20%
- => 2/1 NTMs have a much stronger impact than 3/2 = $-\infty$

1 Overview

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Minimal resistive MHD equations

- Equilibrium: no plasma flow, $B_{eq}(x) = B_0 = \psi'_0(x)$, $\mathbf{j}_{eq}(x) = \psi''_0(x)$, $\psi_0(-x) = \psi_0(x)$, $\rho_0 = \text{Constant}$ (uncompressible)
- Perturbation:

$$\mathsf{B}(x,y,t) = \mathbf{\hat{z}} imes
abla \psi =
abla imes (-\psi \mathbf{\hat{z}}), \ \mathbf{v}(x,y,t) = \mathbf{\hat{z}} imes
abla \phi$$

Linearized MHD equations

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{\eta}{\mu_0} \Delta \mathbf{B}$$
(1)
$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla \rho + \mu_0^{-1} ((\nabla \times \mathbf{B}) \times \mathbf{B}_0 + (\nabla \times \mathbf{B}_0) \times \mathbf{B})$$
(2)
$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{v} = 0$$
(3)

Consider perturbations $\psi(x, y, t) = \psi(x) \exp(\gamma t + iky)$ with $k = mk_1 = m\frac{2\pi}{L_y}$. Taking (1) \cdot **x** and $\nabla \times$ (2) \cdot **2**, gives ...

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Linearized MHD equations

$$\gamma B_x = ik B_{0y} V_x + \frac{\eta}{\mu_0} (\frac{d^2}{dx^2} - k^2) B_x$$
 (4)

$$\rho_0 \gamma (\frac{d^2}{dx^2} - k^2) V_x = \frac{ikB_{0y}}{\mu_0} (\frac{d^2}{dx^2} - k^2 - \frac{B_{0y}'}{B_{0y}}) B_x$$
(5)

$$-ik\psi = B_x$$
 and $-ik\phi = V_x$ (6)

Characteristic times:

- Alfvén time: $\tau_A = a/V_A$ where $V_A = \frac{B_0}{\sqrt{\mu_0\rho}}$ is the Alvén velocity, B_0 is a typical magnetic field amplitude of the system

- Resistive time:
$$\tau_R = \frac{\mu_0 a}{\eta}$$

- Lundquist Number: $S= au_R/ au_Approx 10^8$ in tokamaks,

 $S \approx 10^{11}$ in the earth magnetosphere

Normalisations:

$$\begin{aligned} -\bar{x} &= x/a, \ \bar{k} = ak, \ \bar{\gamma} = \gamma/\tau_A, \ F(\bar{x}) = B_{0y}/B_0, \ F' = dF/d\bar{x} \\ -\psi_N(\bar{x}) &= \psi/\psi_0 = B_x/B_0, \ \phi_N(\bar{x}) = \frac{B_0}{\psi_0}\frac{ik\phi}{\gamma^{-1}}. \end{aligned}$$

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The ideal solution is singular at the resonance

• Linearized MHD equations $(\psi_N
ightarrow \psi, \phi_N
ightarrow \phi)$

$$\bar{\gamma}(\psi - F\phi) = S^{-1}(\frac{d^2}{dx^2} - k^2)\psi$$
(7)
$$\bar{\gamma}^2(\frac{d^2}{dx^2} - k^2)\phi = -\bar{k}^2F(\frac{d^2}{d\bar{x}^2} - \bar{k}^2 - \frac{F''}{F})\psi$$
(8)
$$F(\bar{x}) = B_{0y}/B_0,$$
(9)

• Characteristic time for the tearing instability: $\tau_A \ll \gamma^{-1} \ll \tau_R \Leftrightarrow \bar{\gamma} \ll 1 \ll S\bar{\gamma}$ $\psi - F\phi = 0$ (Flux freezing constraint) $(\frac{d^2}{d\bar{z}^2} - \bar{k}^2 - \frac{F''}{F})\psi = 0$ ($\nabla_{\parallel} j = 0$ or $\nabla \times (\mathbf{J} \times \mathbf{B}) = 0$)(10)

The last equation correspond to a static force balanced! ...

• Ideal MHD breaks down at the resonance where $F(x) \rightarrow 0$: $\phi = \psi/F$ becomes singular.

Magnetic field discontinuity in the ideal limit: Δ

 The resistivity smooths the solution in a resistive layer around the resonance...

$$(\frac{d^2}{d\bar{x}^2} - \bar{k}^2 - \frac{F''}{F})\psi = 0$$
 (11)

- $a\Delta' = \bar{\Delta}' = \lim_{\epsilon \to 0} \frac{\psi'(\epsilon/2) \psi'(-\epsilon/2)}{\psi(0)}$ caraterizes the ideal discontinuity (Warning: $\Delta' \neq \Delta'(\psi_X)$)
- Consider $F(x) = B_{0y}/B_0$, = tanh(x/a) with B.C $\psi(\pm \infty) = 0$, then Eq.(11) can be solved and

$$\Delta' = \frac{2}{a} (\frac{1}{ak} - ak)$$

 The resistive solution will be unstable if and only if Δ '>0. Here, Δ' > 0 if ak ≤ 1 (small wave numbers as expected).

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Ideal structure of the perturbation $\psi(x, y, t) = \psi(x) \exp(\gamma t + iky)$



Right: $\Delta' < 0$

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• To obtain an island one has to draw $\psi_{ ext{tot}} = \psi_0 + \psi$

To solve ∇_{||}j = 0 for any equilibrium, one can use a shooting method: ψ''(x ↔ t) = α(t)ψ(t) see blackboard

The resistive solution around the resonance

- The resistivity smooths the solution in a resistive layer around the resonance...
- Small island approximation: $kx \le kw \ll 1 \Rightarrow \frac{d}{dx} \gg k$ and $F(\bar{x}) \approx \bar{x}$ (Taylor expansion with F(0) = 0, <u>a is now fixed and is a magnetic shear length</u>)

$$\bar{\gamma}(\psi - \bar{x}\phi) = S^{-1}\frac{d^2\psi}{dx^2}$$
(12)
$$\bar{\gamma}^2\frac{d^2\phi}{dx^2} = -\bar{k}^2\bar{x}\frac{d^2\psi}{d\bar{x}^2}$$
(13)
$$F(\bar{x}) = B_{0y}/B_0,$$
(14)

- Constant- ψ approximation in the resistive layer: ψ =Ct but $\psi' \neq 0$

$$ar{x}ar{k} + rac{ar{x}^2ar{k}^2}{\psi\gamma}\phi(ar{x}) = rac{1}{\psi S}\phi''(ar{x})$$

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The resistive solution: matching

• We introduce z = x/r, $\phi(x) = s\chi(z)$, $\phi'' = \frac{s}{r^2}\chi''(x)$ which gives

$$z+z^2\chi(z)=\chi''(z)$$

if $s = -\frac{\psi \bar{\gamma}}{kr}$ and $r^4 = \frac{\bar{\gamma}}{k^2}$ Z! The equation for χ does not depends on the physical parameters and is localized

• The matching condition isa

$$\bar{\Delta}' = \lim_{\varepsilon \to \delta_h} \frac{\psi'(\varepsilon/2) - \psi'(-\varepsilon/2)}{\psi} = \frac{1}{\psi} \lim_{\varepsilon \to \delta_h} \int_{-\varepsilon/2}^{+\varepsilon/2} \psi''(x) dx$$
(15)

It gives $\bar{\Delta}' = \frac{1}{\psi} f(\bar{\gamma}, \bar{k}, S) \int_{-z_h/2}^{+z_h/2} \frac{\chi''}{z} dz$.

• Using that $\mathscr{A} \equiv \int_{-z_h/2}^{+z_h/2} \frac{\chi^{"}}{z} dz \approx \int_{-\infty/2}^{+\infty} \frac{\chi^{"}}{z} dz \approx 2.12$, one obtains

$$\bar{\Delta}' = \bar{k}^{-1/2} \bar{\gamma}^{5/4} S^{3/4} \mathscr{A}$$
(16)

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• In other words, as $\mathscr{A}^{-4/5}\approx 0.55$ the linear growth rate for the tearing instability is

$$\gamma \tau_A = 0.55 \,\bar{\Delta}^{\prime 4/5} \bar{k}^{2/5} S^{-3/5} \propto \eta^{3/5} \tag{17}$$

• The width of the resistive layer is $r=\delta_n \propto \eta^{2/5}$

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- Magnetic islands are resistive structures growing on low order rationnal surfaces
- When the width of the island is large enough, pressure flattening occurs
- The degradation of the confinement is proportionnal to the width
- NTM are magnetic islands amplified by neoclassical mechanisms and should be metastable on ITER baseline scenario
- NTMs and more generally, MHD resistive activity can be the cause of a disruption
- NTMs are seeded by MHD activity and, also, potentially, by turbulence

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